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Suzuki type fixed point theorems for generalized multi-valued mappings on a set endowed with two *b*-metrics

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ABSTRACT

In this paper, we obtained Suzuki type fixed point results for a generalized multi-valued mapping on a set equipped with two *b*-metrics. As a consequence, existence and uniqueness of solution of functional equation arising in dynamical programming is also derived.

1. Introduction and preliminaries

Let (X,d) be a metric space and $CB(X)$ be a family of all nonempty closed and bounded subsets of X. A Hausdorff metric H induced by the metric d of X is given by

$$H(A,B)=\max\left\{\sup_{x\in A}d(x,B),\sup_{y\in B}d(y,A)\right\}$$

for every $A,B\in CB(X)$. A multi-valued mapping $T:X\to CB(X)$ is said to be a *contraction* if there exists a constant $k\in[0,1)$ such that for any $x,y\in X$,

$$H(Tx,Ty)\leqslant kd(x,y).$$

A point $x\in X$ is called a fix d point of T if $x\in Tx$. In 1969 Nadler [19] obtained the following multi-valued version of Banach contraction principle.

Theorem 1.1. *Let (X,d) be a complete metric space and $T:X\to CB(X)$ be a contraction. Then T has a fixed point.*

Nadler's multi-valued contraction theorem [19] (see also [9]) was subsequently generalized among others by [6–8,21,17,11,22,15,16]. Recently Djorić and Lazović [12] proved the following Suzuki type fixed point theorem.

Theorem 1.2 [12]. *Let (X,d) is a complete metric space and $T:X\to CB(X)$. If there exist $0\leqslant r<1$ and a non-increasing function φ from $[0,1)$ into $(0,1]$ defined by*

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